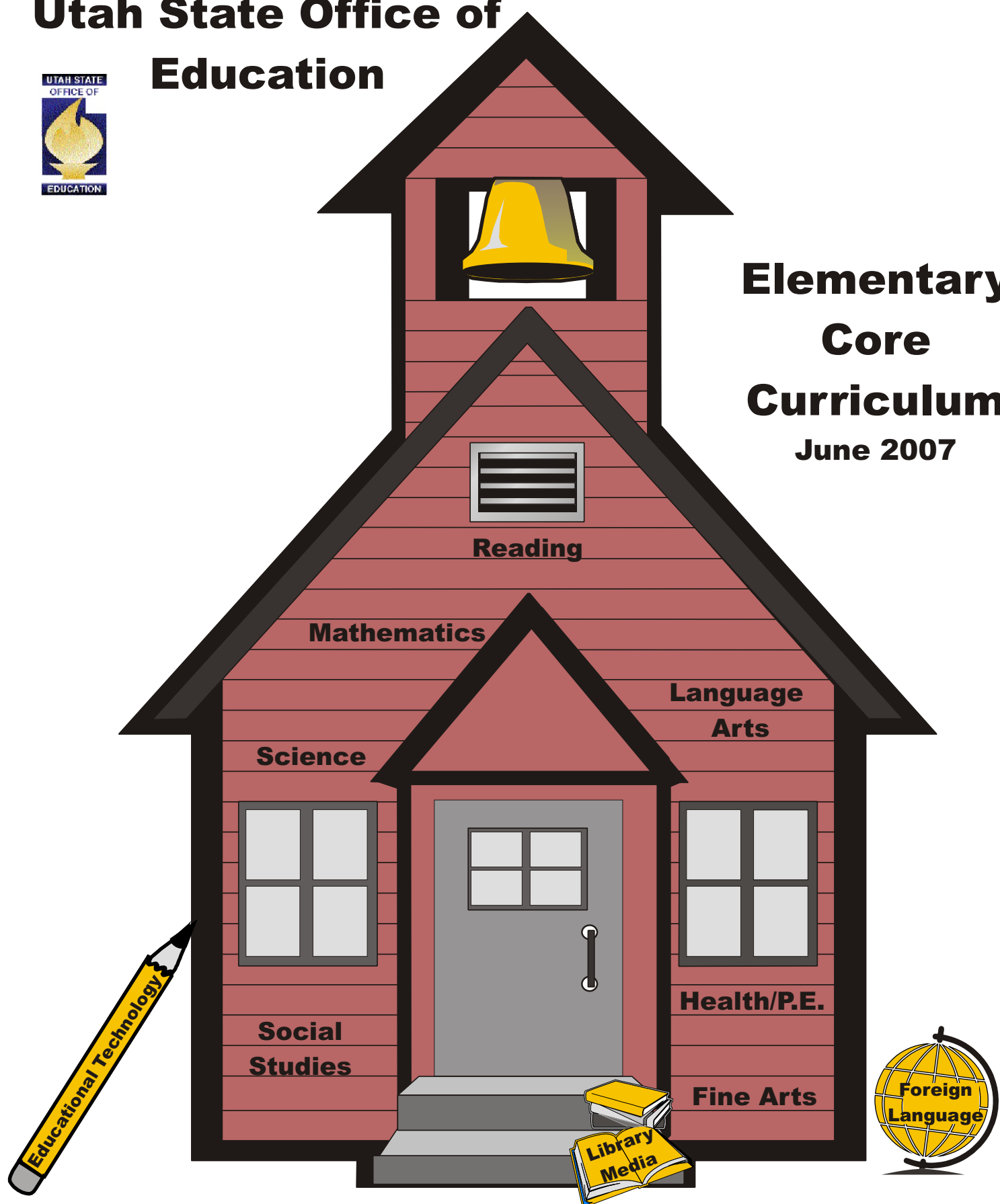


Utah State Office of Education



Elementary Core Curriculum

June 2007



Mathematics K-6

ELEMENTARY CORE CURRICULUM

MATHEMATICS K-6

UTAH STATE OFFICE OF EDUCATION

Patti Harrington
State Superintendent of Public Instruction

Brenda Hales, Associate Superintendent
Student Achievement and School Success

Brett D. Moulding, Director
Curriculum and Instruction

Lynne Greenwood, Coordinator
Curriculum and Instruction

Gerolynn Hargrove, Coordinator
Curriculum and Instruction

Nicole Paulson, Specialist
Mathematics K-6

June 2007

UTAH STATE BOARD OF EDUCATION

250 East 500 South
P. O. Box 144200
Salt Lake City, UT 84114-4200

District 1

Teresa L. Theurer
66 Canterbury Circle
Logan, UT 84321
Phone: (435) 753-0740

District 2

Greg W. Haws
5841 West 4600 South
Hooper, UT 84315
Phone: (801) 985-7980

District 3

Richard Moss
3514 E. Fairway Circ.
Spanish Fork, UT 84660
Phone: (801) 787-1676

District 4

Richard Sadler
875 Edgewood Dr.
Ogden, UT 84403
Phone: (801) 479-7988

District 5

Kim R. Burningham
932 Canyon Crest Drive
Bountiful, UT 84010
Phone: (801) 292-9261

Josh M. Reid

Board of Regents
201 S. Main, Suite 1800
Ogden, UT 84403
Phone: (801) 536-6787

District 6

Michael G. Jensen
4139 S. Aubrey Lane
West Valley City, UT 84128
Phone: (801) 968-5960

District 7

Randall Mackey
1172 E. 100 S.
Salt Lake City, UT 84102
Phone: (801) 575-5000

District 8

Janet A. Cannon
5256 Holladay Blvd.
Salt Lake City, UT 84117
Phone: (801) 272-3516

District 9

Denis Morrill
6024 S. 2200 W.
Taylorsville, UT 84118
Phone: (801) 969-2334

District 10

Laurel Brown
5311 South Lucky Clover Ln
Murray, UT 84123
Phone: (801) 261-4221

Rosanita Cespedes

Board of Regents
1470 S. 400 E.
Salt Lake City, UT 84115
Phone: (801) 466-7371

District 11

Bill Colbert
14866 Village Vista Dr.
Draper, UT 84020
Phone: (801) 572-1608

District 12

Mark Cluff
645 West Hubbard Cir
Alpine, UT 84004
Phone: (801) 756-7623

District 13

Thomas Gregory
1056 W. 1150 S.
Provo, UT 84601
Phone: (801) 607-4702

District 14

Dixie Allen
218 West 5250 North
Vernal, UT 84078
Phone: (435) 789-0534

District 15

Debra G. Roberts
Box 1780
Beaver, UT 84713
Phone: (435) 438-5843

Cyndee Miya

*Coalition of Minorities
Advisory Committee*
218 West 5250 North
Vernal, UT 84078
Phone: (435) 789-0534

Patti Harrington
Executive Officer

Twila B. Affleck
Secretary

Table of Contents

Introduction.....	vii
R277-700-The Elementary and Secondary School Core Curriculum.....	ix
Utah Elementary Mathematics Core Curriculum	1
Guidelines Used in Developing the Elementary Math Core.....	3
Organization of the Elementary Mathematics Core.....	5
Kindergarten Mathematics.....	7
First Grade Mathematics.....	11
Second Grade Mathematics	15
Intended Learning Outcomes for Third Through Sixth Grade Mathematics.....	19
Third Grade Mathematics	21
Fourth Grade Mathematics	27
Fifth Grade Mathematics... ..	33
Sixth Grade Mathematics.....	55

INTRODUCTION

Action by the Utah State Board of Education in January 1984 established a policy requiring the identification of specific Core Curriculum standards, which must be completed by all students K-12 as a requisite for graduation from Utah's secondary schools. This action was followed by three years of extensive work involving all levels of the education family in the process of identifying, trial testing, and refining these Core Curriculum standards for Utah's schools.

The Core Curriculum represents those standards of learning that are essential for all students. They are the ideas, concepts, and skills that provide a foundation on which subsequent learning may be built.

The Core should be taught with respect for differences in learning styles, learning rates, and individual capabilities without losing sight of the common goals. Although the Core Curriculum standards are intended to occupy a major part of the school program, they are not the total curriculum of a level or course.

R277. Education, Administration.

R277-700. The Elementary and Secondary School Core Curriculum.

R277-700-1. Definitions.

A. "Accredited" means evaluated and approved under the Standards for Accreditation of the Northwest Association of Schools and Colleges or the accreditation standards of the Board, available from the USOE Accreditation Specialist.

B. "Applied technology education (ATE)" means organized educational programs or courses which directly or indirectly prepare students for employment, or for additional preparation leading to employment, in occupations, where entry requirements generally do not require a baccalaureate or advanced degree.

C. "Basic skills course" means a subject which requires mastery of specific functions and was identified as a course to be assessed under Section 53A-1-602.

D. "Board" means the Utah State Board of Education.

E. "Core Curriculum content standard" means a broad statement of what students enrolled in public schools are expected to know and be able to do at specific grade levels or following completion of identified courses.

F. "Core Curriculum criterion-referenced test (CRTs)" means a test to measure performance against a specific standard. The meaning of the scores is not tied to the performance of other students.

G. "Core Curriculum objective" means a more focused description of what students enrolled in public schools are expected to know and do at the completion of instruction.

H. "Demonstrated competence" means subject mastery as determined by school district standards and review. School district review may include such methods and documentation as: tests, interviews, peer evaluations, writing samples, reports or portfolios.

I. "Elementary school" for purposes of this rule means grades K-6 in whatever kind of school the grade levels exist.

J. "High school" for purposes of this rule means grades 9-12 in whatever kind of school the grade levels exist.

K. "Individualized Education Program (IEP)" means a written statement for a student with a disability that is developed, reviewed, and revised in accordance with the Utah Special Education Rules and Part B of the Individuals with Disabilities Education Act (IDEA).

L. "Middle school" for purposes of this rule means grades 7-8 in whatever kind of school the grade levels exist.

M. "Norm-referenced test" means a test where the scores are based on comparisons with a nationally representative group of students in the same grade. The meaning of the scores is tied specifically to student performance relative to the performance of the students in the norm group under very specific testing conditions.

N. "State core Curriculum (Core Curriculum)" means those standards of learning that are essential for all Utah students, as well as the ideas, concepts, and skills that provide a foundation on which subsequent learning may be built, as established by the Board.

O. "USOE" means the Utah State Office of Education.

P. "Utah Basic Skills Competency Test" means a test to be administered to Utah students beginning in the tenth grade to include at a minimum components on English, language arts, reading and mathematics. Utah students shall satisfy the requirements of the Utah Basic Skills Competency Test in addition to school or district graduation requirements prior to receiving a basic high school diploma.

R277-700-2. Authority and Purpose.

A. This rule is authorized by Article X, Section 3 of the Utah Constitution, which places general control and supervision of the public schools under the Board; Section 53A-1-402(1)(b) and (c) which directs the Board to make rules regarding competency levels, graduation requirements, curriculum, and instruction requirements; Section 53A-1-402.6 which directs the Board to establish a Core Curriculum in consultation with local boards and superintendents and directs local boards to design local programs to help students master the Core Curriculum; and Section 53A-1-401(3) which allows the Board to adopt rules in accordance with its responsibilities.

B. The purpose of this rule is to specify the minimum Core Curriculum requirements for the public schools, to give directions to local boards and school districts about providing the Core Curriculum for the benefit of students, and to establish responsibility for mastery of Core Curriculum requirements.

R277-700-3. Core Curriculum Standards and Objectives.

A. The Board establishes minimum course description standards and objectives for each course in the required

general core, which is commonly referred to as the Core Curriculum.

B. Course descriptions for required and elective courses shall be developed cooperatively by school districts and the USOE with opportunity for public and parental participation in the development process.

C. The descriptions shall contain mastery criteria for the courses, and shall stress mastery of the course material and Core objectives and standards rather than completion of predetermined time allotments for courses.

D. Implementation of the Core Curriculum and student assessment procedures are the responsibility of local boards of education consistent with state law.

E. This rule shall apply to students in the 2005-2006 graduating class.

R277-700-4. Elementary Education Requirements.

A. The Board shall establish a Core Curriculum for elementary schools, grades K-6.

B. Elementary School Education Core Curriculum Content Area Requirements:

- (1) Grades K-2:
 - (a) Reading/Language Arts;
 - (b) Mathematics;
 - (c) Integrated Curriculum.
- (2) Grades 3-6:
 - (a) Reading/Language Arts;
 - (b) Mathematics;
 - (c) Science;
 - (d) Social Studies;
 - (e) Arts:
 - (i) Visual Arts;
 - (ii) Music;
 - (iii) Dance;
 - (iv) Theatre.
 - (f) Health Education;
 - (g) Physical Education;
 - (h) Educational Technology;
 - (i) Library Media.

C. It is the responsibility of the local boards of education to provide access to the Core Curriculum to all students.

D. Student mastery of the general Core Curriculum is the responsibility of local boards of education.

E. Informal assessment should occur on a regular basis to ensure continual student progress.

F- Board-approved CRT's shall be used to assess student mastery of the following:

- (1) reading;
- (2) language arts;
- (3) mathematics;
- (4) science in elementary grades 4-6; and
- (5) effectiveness of written expression.

G. Norm-referenced tests shall be given to all elementary students in grades 3 and 5.

H. Provision for remediation for all elementary students who do not achieve mastery is the responsibility of local boards of education.

R277-700-5. Middle School Education Requirements.

A. The Board shall establish a Core Curriculum for middle school education.

B. Students in grades 7-8 shall earn a minimum of 12 units of credit to be properly prepared for instruction in grades 9-12.

C. Local boards may require additional units of credit.

D. Grades 7-8 Core Curriculum Requirements and units of credit:

- (1) General Core (10.5 units of credit):
 - (a) Language Arts (2.0 units of credit) ;
 - (b) Mathematics (2.0 units of credit);
 - (c) Science (1.5 units of credit);
 - (d) Social Studies (1.5 units of credit);
 - (e) The Arts (1.0 units of credit):
 - (i) Visual Arts;
 - (ii) Music;
 - (iii) Dance;
 - (iv) Theatre.
 - (f) Physical Education (1.0 units of credit);
 - (g) Health Education (0.5 units of credit);
 - (h) Applied Technology Education Technology, Life, and Careers (1.0 units of credit);
 - (i) Educational Technology (credit optional);
 - (j) Library Media (integrated into subject areas).

E. Board-approved CRT's shall be used to assess student mastery of the following:

- (1) reading;
- (2) language arts;
- (3) mathematics;
- (4) science in grades 7 and 8; and
- (5) effectiveness of written expression.

F. Norm-referenced tests shall be given to all middle school students in grade 8.

R277-700-6. High School Requirements.

A. The Board shall establish a Core Curriculum for students in grades 9-12.

B. Students in grades 9-12 shall earn a minimum of 24 units of credit.

C. Local boards may require additional units of credit.

D. Grades 9-12 Core Curriculum requirements required units of credit:

(1) Language Arts (3.0 units of credit);

(2) Mathematics (2.0 units of credit):

(a) minimally, Elementary Algebra or Applied Mathematics I; and

(b) geometry or Applied Mathematics II; or

(c) any Advanced Mathematics courses in sequence beyond (a) and (b) ;

(d) high school mathematics credit may not be earned for courses in sequence below (a).

(3) Science (2.0 units of credit from two of the four science areas):

(a) earth science (1.0 units of credit);

(b) biological science (1.0 units of credit);

(c) chemistry (1.0 units of credit);

(d) physics (1.0 units of credit).

(4) Social Studies (3.0 units of credit):

(a) Geography for Life (0.5 units of credit);

(b) World Civilizations (0.5 units of credit);

(c) U.S. history (1.0 units of credit);

(d) U.S. Government and Citizenship (0.5 units of Credit);

(e) elective social studies class (0.5 units of

(5) The Arts (1.5 units of credit from any of the following performance areas):

(a) visual arts;

(b) music;

(c) dance;

(d) theatre;

(6) Health education (0.5 units of credit)

(7) Physical education (1.5 units of credit):

(a) participation skills (0.5 units of credit);

(b) Fitness for Life (0.5 units of credit);

- (c) individualized lifetime activities (0.5 units of credit) or team sport/athletic participation (maximum of 0.5 units of credit with school approval).
- (8) Applied technology education (1.0 units of credit);
 - (a) agriculture;
 - (b) business;
 - (c) family and consumer sciences;
 - (d) technology education;
 - (h) trade and technical education.
- (9) Educational technology:
 - (a) computer Technology (0.5 units of credit for the class by this specific name only); or
 - (b) successful completion of state-approved competency examination (no credit, but satisfies the Core requirement).
- (10) Library media skills integrated into the curriculum;
- (11) Board-approved CRT's shall be used to assess student mastery of the following subjects:
 - (a) reading;
 - (b) language arts through grade 11;
 - (c) mathematics as defined under R277-700-6D(2);
 - (d) science as defined under R277-700-6D(3); and
 - (e) effectiveness of written expression.
- E. Students shall participate in the Utah Basic Skills Competency Test, as defined under R277-700-10.
- F. Students with disabilities served by special education programs may have changes made to graduation requirements through individual IEPs to meet unique educational needs. A student's IEP shall document the nature and extent of modifications, substitutions or exemptions made to accommodate a student with disabilities.

R277-700.7. Student Mastery and Assessment of Core Curriculum Standards and Objectives.

- A. Student mastery of the Core Curriculum at all levels is the responsibility of local boards of education.
- B. Provisions for remediation of secondary students who do not achieve mastery is the responsibility of local boards of education under Section 53A-13-104.
- C. Students who are found to be deficient in basic skills through U-PASS shall receive remedial assistance according to provisions of Section 53A-1-606(1).

D. If parents object to portions of courses or courses in their entirety under provisions of law (Section 53A-13-101.2) and rule (R277-105), students and parents shall be responsible for the mastery of Core objectives to the satisfaction of the school prior to promotion to the next course or grade level.

E. Students with Disabilities:

(1) All students with disabilities served by special education programs shall demonstrate mastery of the Core Curriculum.

(2) If a student's disabling condition precludes the successful demonstration of mastery, the student's IEP team, on a case-by-case basis, may provide accommodations for or modify the mastery demonstration to accommodate the student's disability.

F. Students may demonstrate competency to satisfy course requirements consistent with R277-705-3.

G. All Utah public school students shall participate in state-mandated assessments, as required by law.

KEY: curricula

March 5, 2002

**Art X Sec 3
53A-1-402(1)(b
53A-1-402.6
53A-1-401(3**

Utah Elementary Mathematics Core Curriculum

Introduction

Most children enter school confident in their own abilities; they are curious and eager to learn more. They make sense of the world by reasoning and problem solving. Young students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematical learners. Students use mathematical tools, such as manipulative materials and technology, to develop conceptual understanding and solve problems as they do mathematics. Students, as mathematicians, learn best through participatory experiences throughout the instruction of the mathematics curriculum.

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, the term *mathematical proficiency* has been chosen to capture what it means to learn mathematics successfully. Mathematical proficiency has five strands: computing (carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately), understanding (comprehending mathematical concepts, operations, and relations), applying (ability to formulate, represent, and solve mathematical problems), reasoning (logically explaining and justifying a solution to a problem), and engaging (seeing mathematics as sensible, useful, and doable, and being able to do the work) (NRC, 2001).

The most important observation about the five strands of mathematical proficiency is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal. At any given moment during a mathematics lesson or unit, one or two strands might be emphasized. But all the strands must eventually be addressed so that the links among them are strengthened. The integrated and balanced development of all five strands of mathematical proficiency should guide the teaching and learning of school mathematics. Instruction should not be based on the extreme positions that students learn solely by internalizing what a teacher or book says, or solely by inventing mathematics on their own.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed and revised by a community of Utah mathematics teachers, mathematicians, university mathematics educators, and State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.

Guidelines Used in Developing the Elementary Mathematics Core

The Core is:

Consistent With the Nature of Learning

In the early grades, children are forming attitudes and habits for learning. It is important that instruction maximizes students' potential and gives them understanding of the intertwined nature of learning. The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world. The Core is designed to produce an integrated set of Intended Learning Outcomes for students.

Coherent

The Core has been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts and skills. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

Developmentally Appropriate

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on providing experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

Reflective of Successful Teaching Practices

Learning through play, movement, and adventure is critical to the early development of the mind and body. The Core emphasizes student exploration. The Core is designed to encourage a variety of interactive learning opportunities. Instruction should include recognition of the role of mathematics in the classroom, school, and community.

Comprehensive

By emphasizing depth rather than breadth, the Elementary Mathematics Core seeks to empower students by providing a comprehensive background in mathematics. Teachers are expected to teach all the standards and objectives specified in the Core for their grade level, but may add related concepts and skills.

Feasible

Teachers and others who are familiar with Utah students, classrooms, teachers, and schools have designed the Core. It can be taught with easily obtained resources and materials. A handbook is also available for teachers and has sample lessons on each topic for each grade level. The handbook is a document that will grow as teachers add exemplary lessons aligned with the new Core.

Useful and Relevant

This curriculum relates directly to student needs and interests. The relevance of mathematics to other endeavors enables students to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom.

Reliant Upon Effective Assessment Practices

Student achievement of the standards and objectives in this Core is best assessed using a variety of assessment instruments. Performance tests are particularly appropriate to evaluate student mastery of mathematical processes and problem-solving skills. Teachers should use a variety of classroom assessment approaches in conjunction with standard assessment instruments to inform instruction. Sample test items, keyed to each Core Standard, may be located on the “Utah Mathematics Home Page” at <http://www.usoe.k12.ut.us/curr/math>. Observation of students engaged in instructional activities is highly recommended as a way to assess students’ skills as well as attitudes toward learning. The nature of the questions posed by students provides important evidence of their understanding of mathematics.

Based Upon the National Council of Teachers of Mathematics Curriculum Focal Points

In 2006, the National Council of Teachers of Mathematics (NCTM) published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006). This document is available online at <http://www.nctm.org/focalpoints>. This document describes three focal points for each grade level. NCTM’s focal points are areas of emphasis recommended for the curriculum of each grade level. The focal points within a grade are *not* the entire curriculum for that particular grade; however, Utah’s Core Curriculum was designed to include these areas of focus.

Organization of the Elementary Mathematics Core

The Core is designed to help teachers organize and deliver instruction.

- Each grade level begins with a brief description of areas of instructional emphasis which can serve as organizing structures for curriculum design and instruction.
- The INTENDED LEARNING OUTCOMES (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are found at the beginning of each grade level and are an integral part of the Core.
- A STANDARD is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- An OBJECTIVE is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard at that grade level. Several Indicators are described for each Objective.
- INDICATORS are observable or measurable student actions that enable students to master an Objective. Indicators can help guide classroom instruction.
- MATHEMATICAL LANGUAGE AND SYMBOLS STUDENTS SHOULD USE includes language and symbols students should use in oral and written language.
- EXPLORATORY CONCEPTS AND SKILLS are included to establish connections with learning in subsequent grade levels. They are not intended to be assessed at the grade level indicated.

Kindergarten Mathematics

By the end of kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense of patterns. Students also develop an understanding of measurable attributes of objects.

Standard 1: Students will understand simple number concepts and relationships.

Objective 1: Identify and use whole numbers up to 30.

- a. Represent whole numbers using concrete, pictorial, and symbolic representations.
- b. Order a set of up to ten objects and use ordinal numbers from first to tenth to identify the position of the object in the chosen order.
- c. Use one-to-one correspondence when counting a set of objects and develop a strategy for keeping track of counted and uncounted objects.

Objective 2: Identify and use simple relationships among whole numbers up to 30.

- a. Estimate quantities in a set of objects using multiples of 10 as benchmark numbers.
- b. Compose and decompose quantities to establish a relationship between the parts and the whole.
- c. Recognize 5 or 10 as a part of the part-whole relationship of numbers.
- d. Compare sets of objects and determine whether they have the same, fewer, or more objects.

Objective 3: Model, describe, and illustrate meanings of addition and subtraction for whole numbers less than ten.

- a. Demonstrate the joining and separating of sets of objects to solve problems.
- b. Describe the joining or separating of sets with informal language when using models.
- c. Record pictorially the results from joining or separating of sets.

Mathematical Language and Symbols Students Should Use

add, subtract, first, second, third, fourth, fifth, sixth, seventh, eighth, ninth, tenth, same, fewer, more

Exploratory Concepts and Skills

- ✓ Count by ones, beginning from any number in the counting sequence.
- ✓ Represent quantities using concrete objects and investigate partitioning of sets.
- ✓ Create problems that can be solved using addition and subtraction.

Standard 2: Students will sort and classify objects as well as recognize and create simple patterns.

Objective 1: Identify, sort, and classify objects according to common attributes.

- a. Sort objects into groups by attribute and identify which attribute was used.
- b. Describe multiple ways to sort and classify a group of objects.

Objective 2: Identify, duplicate, describe, and extend simple repeating and growing patterns.

- a. Identify and describe simple repeating patterns with numbers and shapes.
- b. Duplicate and extend simple repeating patterns with numbers and shapes.
- c. Describe simple growing patterns with shapes.
- d. Identify simple patterns in the environment.

Mathematical Language and Symbols Students Should Use

sort, repeating patterns, growing patterns

Exploratory Concepts and Skills

- ✓ Explore skip counting by fives, tens, and twos.

Standard 3: Students will understand basic geometry and measurement concepts as well as collect and organize data.

Objective 1: Identify and create simple geometric shapes and describe simple spatial relationships.

- a. Identify, name, describe, and draw circles, triangles, rectangles, and squares in various sizes and orientations.
- b. Combine shapes to create two-dimensional objects (e.g., using a triangle and square to create a picture of a house).
- c. Use words to describe position and distance.
- d. Investigate two- and three-dimensional shapes including hexagons, trapezoids, spheres, cubes, and cones.

Objective 2: Identify and use measurable attributes of objects and units of measurement.

- a. Identify clocks and calendars as tools that measure time.
- b. Identify a day, week, and month on a calendar and name the days of the week in order.
- c. Identify pennies, nickels, dimes, and quarters as units of money.
- d. Compare two objects by measurable attributes (i.e., length, weight) and order several objects by measurable attributes (i.e., length, weight).

Objective 3: Collect and organize simple data.

- a. Pose questions and gather data about self and surroundings.
- b. Organize data obtained from sorting and classifying objects.

Mathematical Language and Symbols Students Should Use

circle, triangle, rectangle, square, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, penny, nickel, dime, quarter, shorter, longer, above, below, near, far, between

Exploratory Concepts and Skills

- ✓ Measure objects using non-standard units.
- ✓ Identify the value of a penny, nickel, dime, and quarter.
- ✓ Organize data in lists, tables, and simple graphs.

First Grade Mathematics

By the end of grade one, students understand and use the concept of ones and tens in the base-ten number system. Students understand the meaning of addition and subtraction and add and subtract small numbers with ease. They measure with simple units and extend their understanding of geometric figures in their environment. They represent, describe, and interpret data and analyze and solve simple problems.

Standard 1: Students will acquire number sense and perform simple operations with whole numbers.

Objective 1: Represent and use whole numbers up to 100.

- a. Count, read, and write whole numbers.
- b. Represent whole numbers using the number line, models, and number sentences.
- c. Represent whole numbers greater than 10 in groups of tens and ones using objects, pictures, and expanded notation.

Objective 2: Identify simple relationships among whole numbers up to 100.

- a. Compare and order sets of objects and numbers using the terms greater than, less than, and equal to when describing the comparisons.
- b. Make reasonable estimates of the quantitative difference between two sets of objects.
- c. Identify one more, one less, 10 more, and 10 less than a given number.
- d. Identify numbers missing from a counting sequence.
- e. Represent part-whole relationships using the number line.

Objective 3: Model, describe, and illustrate the meanings of addition and subtraction and use these operations to solve problems.

- a. Use a variety of models, including objects, length-based models, the number line and the ten frame to describe problem types (i.e., part-whole, combine, separate, compare).
- b. Use the properties of addition (i.e., commutativity, associativity, identity element) and the mathematical relationship between addition and subtraction to solve problems.
- c. Compute basic addition facts (up to $10 + 10$) and the related subtraction facts using strategies (e.g., $6 + 7 = (6 + 4) + 3 = 10 + 3 = 13$).
- d. Find the sum of three one-digit numbers.

Mathematical Language and Symbols Students Should Use

add, sum, subtract, difference, greater than, less than, equal to

Exploratory Concepts and Skills

- ✓ Use concrete materials to investigate situations that lead to multiplication and division.
- ✓ Develop and use strategies for addition and subtraction of multi-digit whole numbers.
- ✓ Investigate the meaning of fraction concepts.
- ✓ Understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.

Standard 2: Students will identify and use number patterns and properties to describe and represent mathematical relationships.

Objective 1: Recognize, describe, and represent patterns with more than one attribute.

- a. Sort and classify objects using more than one attribute.
- b. Identify, create, and label repeating patterns using objects, pictures, and symbolic notation.
- c. Identify, create, and label growing patterns using objects, pictures, and symbolic notation.
- d. Use patterns to establish skip counting by twos, fives, and tens.

Objective 2: Recognize and represent mathematical relationships using symbols and use number sentences with operational symbols to solve problems.

- a. Recognize that “=” indicates that the two sides of an equation are expressions of the same number.
- b. Recognize that “+” indicates the joining of sets and that “-” indicates the separation of sets.
- c. Write and solve number sentences from problem situations involving addition and subtraction, using symbolic notation for the missing value (e.g., $\Delta + 4 = 7$).
- d. Create problem situations from given number sentences involving addition and subtraction.

Mathematical Language and Symbols Students Should Use

sort, attribute, repeating patterns, growing patterns, skip count, number sentence, symbol, +, -, =

Exploratory Concepts and Skills

- ✓ Investigate situations with variables as unknowns and as quantities that vary.

Standard 3: Students will understand simple geometry and measurement concepts as well as collect, represent, and draw conclusions from data.

Objective 1: Identify, describe, and create simple geometric figures.

- a. Name, create, and sort geometric plane figures (i.e., circle, triangle, rectangle, square, trapezoid, rhombus, parallelogram, hexagon).
- b. Identify geometric plane and solid figures (i.e., circle, triangle, rectangle, square, trapezoid, hexagon, rhombus, parallelogram, cube, sphere, cone) in the students’ environment.
- c. Compose and decompose plane and solid figures (e.g., make two triangles from a square) and describe the part-whole relationships, the attributes of the figures, and how they are different and similar.

Objective 2: Identify measurable attributes of objects and units of measurement, and use appropriate techniques and tools to determine measurements.

- a. Identify the appropriate tools for measuring length, weight, capacity, temperature, and time.
- b. Measure the length of an object using nonstandard units and count the units using groups of tens and ones.
- c. Identify the value of a penny, nickel, dime, quarter, and dollar, and determine the value of a set of the same coins that total 25¢ or less (e.g., a set of 5 nickels equals 25¢).
- d. Tell time to the hour and half-hour.
- e. Name the months of the year and seasons in order, and use a calendar to determine the day of the week and date.

Objective 3: Collect, organize, and represent simple data.

- a. Collect and represent data using tables, tally marks, pictographs, and bar graphs.
- b. Describe and interpret data.

Mathematical Language and Symbols Students Should Use

circle, triangle, rectangle, square, trapezoid, hexagon, rhombus, parallelogram, cube, sphere, cone, penny, nickel, dime, quarter, dollar, January, February, March, April, May, June, July, August, September, October, November, December, winter, spring, summer, fall, data, value, graph, tally mark

Exploratory Concepts and Skills

- ✓ Compare objects using non-standard units.
- ✓ Interpret data from charts and graphs.

Second Grade Mathematics

By the end of grade two, students understand place value and number relationships in addition and subtraction and they model simple concepts of multiplication and division. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to their geometric attributes. They collect and analyze data and verify the answers.

Standard 1: Students will acquire number sense with whole numbers and fractions and perform operations with whole numbers.

Objective 1: Identify and represent the relationships among numbers, quantities, and place value in whole numbers up to 1000.

- a. Represent whole numbers in groups of hundreds, tens, and ones using base ten models and write the numeral representing the set in standard and expanded form.
- b. Identify the place and the value of a given digit in a three-digit numeral.
- c. Represent the composition and decomposition of numbers in a variety of ways.
- d. Compare and order numbers using the terms, greater than, less than, or equal to, and the symbols, $>$, $<$, and $=$, using various strategies, including the number line.
- e. Identify and describe even and odd whole numbers.

Objective 2: Use unit fractions to identify parts of the whole and parts of a set.

- a. Divide geometric shapes into two, three, or four equal parts and identify the parts as halves, thirds, or fourths.
- b. Divide sets of objects into two, three, or four parts of equal number of objects and identify the parts as halves, thirds, or fourths.
- c. Represent the unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ with objects, pictures, words (e.g., ___ out of ___ equal parts), and symbols.

Objective 3: Estimate, model, illustrate, describe, and solve problems involving two- and three-digit addition and subtraction.

- a. Demonstrate quick recall of addition facts (up to $10 + 10$) and related subtraction facts.
- b. Model addition and subtraction of two- and three-digit whole numbers (sums and minuends to 1000) in a variety of ways.
- c. Write a story problem that relates to a given addition or subtraction equation, and write a number sentence to solve a story problem that is related to the environment.
- d. Demonstrate fluency with two- and three-digit addition and subtraction problems, using efficient, accurate, and generalizable strategies that include standard algorithms and mental arithmetic, and describe why the procedures work.
- e. Use the mathematical relationship between addition and subtraction and properties of addition to model and solve problems.

Objective 4: Model, illustrate, and pictorially record solutions to simple multiplication and division problems.

- a. Represent multiplication with equal groups using concrete objects and skip counting by twos, fives, and tens.
- b. Represent division as fair shares using concrete objects or pictures.

Mathematical Language and Symbols Students Should Use

number line, add, sum, subtract, difference, greater than, less than, equal to, $>$, $<$, $=$, even, odd, halves, thirds, fourths, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

Exploratory Concepts and Skills

- ✓ Investigate addition of common fractions (e.g., $\frac{1}{2} + \frac{1}{2} = 1$, $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$).
- ✓ Investigate comparing fractions in terms of greater than, less than, and equal to.
- ✓ Understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.

Standard 2: Students will model, represent, and interpret patterns and number relationships to create and solve problems with addition and subtraction.

Objective 1: Recognize, describe, create, and extend growing patterns.

- a. Determine the next term in linear patterns (e.g., 2, 4, 6...; the number of hands on one person, two people, three people).
- b. Construct models and skip count by twos, threes, fives, and tens and relate to repeated addition.

Objective 2: Model, represent, and interpret number relationships using mathematical symbols.

- a. Recognize that “ \neq ” indicates a relationship in which the two sides of the inequality are expressions of different numbers.
- b. Recognize that symbols such as x , \triangle , or \diamond in an addition or subtraction equation represent a number that will make the statement true.
- c. Use the commutative and associative properties of addition to simplify calculations.

Mathematical Language and Symbols Students Should Use

patterns, $+$, $-$, $=$, \neq

Exploratory Concepts and Skills

- ✓ Investigate situations with variables as unknowns and as quantities that vary.

Standard 3: Students will understand simple geometry and measurement concepts as well as collect, represent, and draw conclusions from data.

Objective 1: Describe, classify, and create geometric figures.

- a. Describe and classify plane and solid geometric figures (i.e., circle, triangle, rectangle, square, trapezoid, rhombus, parallelogram, pentagon, hexagon, cube, sphere, cone) according to the number of sides and angles or faces, edges, and vertices.
- b. Compose and decompose shapes and figures by substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for arrangements of smaller shapes.
- c. Compose and decompose shapes and figures and describe the part-whole relationships, similarities, and differences.

Objective 2: Identify and use units of measure, iterate (repeat) that unit, and compare the number of iterations to the item being measured.

- a. Identify and use measurement units to measure, to the nearest unit, length (i.e., inch, centimeter), weight in pounds, and capacity in cups.
- b. Estimate and measure length by iterating a nonstandard or standard unit of measure.
- c. Use different units to measure the length of the same object and recognize that the smaller the unit, the more iterations needed to cover a given length.
- d. Determine the value of a set of up to five coins that total \$1.00 or less (e.g., three dimes, one nickel, and one penny equals 36¢).
- e. Tell time to the quarter-hour and sequence a series of daily events by time (e.g., breakfast at 7:00 a.m., school begins at 9:00 a.m, school ends at 3:00 p.m.).

Objective 3: Collect, record, organize, display, and interpret numerical data.

- a. Collect and record data systematically, using a strategy for keeping track of what has been counted.
- b. Organize and represent the same data in more than one way.
- c. Organize, display, and label information, including keys, using pictographs, tallies, bar graphs, and organized tables.
- d. Describe data represented on charts and graphs and answer simple questions related to data representations.

Mathematical Language and Symbols Students Should Use

inch, centimeter, pound, cup, circle, triangle, rectangle, square, trapezoid, rhombus, parallelogram, pentagon, hexagon, cube, sphere, cone, vertices, angle, face, edge, weight, length, capacity

Exploratory Concepts and Skills

- ✓ Use verbal instructions to move within the environment.
- ✓ Determine simple equivalencies of measurements.
- ✓ Conduct simple probability experiments.

Intended Learning Outcomes for Third through Sixth Grade Mathematics

The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for student learning in mathematics.

ILOs for mathematics:

- 1. Develop a positive learning attitude toward mathematics.**
- 2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.**
- 3. Reason logically, using inductive and deductive strategies and justify conclusions.**
- 4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.**
- 5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.**
- 6. Represent mathematical ideas in a variety of ways.**

Significant mathematics understanding occurs when teachers incorporate ILOs in planning mathematics instruction. The following are ideas to consider when planning instruction for students to acquire the ILOs:

1. Develop a positive learning attitude toward mathematics.

When students are confident in their mathematical abilities, they demonstrate persistence in completing tasks. They pose mathematical questions about objects, events, and processes while displaying a sense of curiosity about numbers and patterns. It is important to build on students' innate problem-solving inclinations and to preserve and encourage a disposition that values mathematics.

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.

Problem solving is the cornerstone of mathematics. Mathematical knowledge is generated through problem solving as students explore mathematics. To become effective problem solvers, students need many opportunities to formulate questions and model problem situations in a variety of ways. They should generalize mathematical relationships and solve problems in both mathematical and everyday contexts.

3. Reason logically, using inductive and deductive strategies and justify conclusions.

Mathematical reasoning develops in classrooms where students are encouraged to put forth their own ideas for examination. Students develop their reasoning skills by making and testing mathematical conjectures, drawing logical conclusions, and justifying their thinking in developmentally appropriate ways. Students use models, known facts, and relationships to explain reasoning. As they advance through the grades, students' arguments become more sophisticated.

4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

The ability to express mathematical ideas coherently to peers, teachers, and others through oral and written language is an important skill in mathematics. Students develop this skill and deepen their understanding of mathematics when they use accurate mathematical language to talk and write about what they are doing. When students talk and write about mathematics, they clarify their ideas and learn how to make convincing arguments and represent mathematical ideas verbally, pictorially, and symbolically.

5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Students develop a perspective of the mathematics field as an integrated whole by understanding connections within mathematics. Students should be encouraged to explore the connections that exist with other disciplines and between mathematics and their own experiences.

6. Represent mathematical ideas in a variety of ways.

Mathematics involves using various types of representations including concrete, pictorial, and symbolic models. In particular, identifying and locating numbers on the number line has a central role in uniting all numbers to promote understanding of equivalent representations and ordering. Students also use a variety of mathematical representations to expand their capacity to think logically about mathematics.

Third Grade Mathematics

By the end of grade three, students develop understandings of multiplication and division of whole numbers. They use properties to develop increasingly more sophisticated strategies to solve problems involving basic multiplication and division facts. They relate division to multiplication. Students understand fraction equivalence for simple fractions; they recognize that the size of a fractional part is relative to the size of the whole. They understand meanings of fractions to represent parts of a whole, parts of a set, or distances on a number line. They compare and order simple fractions by using models, benchmark fractions, or common denominators.

Students investigate, analyze, and classify two-dimensional shapes by their sides and angles. They decompose, combine, and transform polygons to understand properties of two-dimensional space and use those properties to solve problems. Students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems.

Standard 1: Students will understand the base-ten numeration system, place value concepts, simple fractions and perform operations with whole numbers.

Objective 1: Represent whole numbers up to 10,000, comprehend place value concepts, and identify relationships among whole numbers using base-ten models and symbolic notation.

- a. Read, write, and represent whole numbers using standard and expanded form.
- b. Demonstrate multiple ways to represent numbers using models and symbolic representations (e.g., fifty is the same as two groups of 25, the number of pennies in five dimes, or $75 - 25$).
- c. Identify the place and the value of a given digit in a four-digit numeral and round numbers to the nearest ten, hundred, and thousand.
- d. Order and compare whole numbers on a number line and use the symbols $<$, $>$, \neq , and $=$ when comparing whole numbers.
- e. Identify factors and multiples of whole numbers.

Objective 2: Use fractions to describe and compare parts of the whole.

- a. Identify the denominator of a fraction as the number of equal parts of the unit whole and the numerator of a fraction as the number of equal parts being considered.
- b. Define regions and sets of objects as a whole and divide the whole into equal parts using a variety of objects, models, and illustrations.
- c. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, sixths, and eighths.
- d. Place fractions on the number line and compare and order fractions using models, pictures, the number line, and symbols.
- e. Find equivalent fractions using concrete and pictorial representations.

Objective 3: Model problems involving addition, subtraction, multiplication, and division.

- a. Demonstrate the meaning of multiplication and division of whole numbers through the use of a variety of representations (e.g., equal-sized groups, arrays, area models, and equal jumps on a number line for multiplication, partitioning and sharing for division).
- b. Use a variety of strategies and tools, such as repeated addition or subtraction, equal jumps on the number line, and counters arranged in arrays to model multiplication and division problems.
- c. Demonstrate, using objects, that multiplication and division by the same number are inverse operations (e.g., $3 \times \square = 12$ is the same as $12 \div 3 = \square$ and $\square = 4$).
- d. Demonstrate the effect of place value when multiplying whole numbers by 10.
- e. Write a story problem that relates to a given addition, subtraction, or multiplication equation, and write a number sentence to solve a problem related to the students' environment.

Objective 4: Compute and solve problems involving addition and subtraction of 3- and 4-digit numbers and basic facts of multiplication and division.

- a. Use a variety of methods to facilitate computation (e.g., estimation, mental math strategies, paper and pencil).
- b. Find the sum or difference of numbers, including monetary amounts, using models and strategies such as expanded form, compensation, partial sums, and the standard algorithm.
- c. Compute basic multiplication facts (0-10) and related division facts using a variety of strategies based on properties of addition and multiplication (i.e., commutative, associative, identity, zero, and the distributive properties).

Mathematical Language and Symbols Students Should Use

sum, difference, expanded form, factor, product, array, multiple, numerator, denominator, halves, thirds, fourths, sixths, eighths, divisor, dividend, quotient, greater than, less than, equal to, $<$, $>$, $=$

Exploratory Concepts and Skills

- ✓ Extend multiplication and division to larger-digit numbers.
- ✓ Use concrete objects and visual models to add and subtract common decimals.
- ✓ Investigate the distributive property of multiplication over addition for single-digit multipliers (e.g., 7×15 is equivalent to $7 \times (10 + 5)$ is equivalent to $(7 \times 10) + (7 \times 5)$).

Standard 2: Students will use patterns, symbols, operations, and properties of addition and multiplication to represent and describe simple number relationships.

Objective 1: Create, represent, and analyze growing patterns.

- a. Create and extend growing patterns using objects, numbers, and tables.
- b. Describe how patterns are extended using manipulatives, pictures, and numerical representations.

Objective 2: Recognize, represent, and simplify simple number relationships using symbols, operations, and properties.

- a. Represent numerical relationships as expressions, equations, and inequalities.
- b. Solve equations involving equivalent expressions (e.g., $6 + 4 = \Delta + 7$).
- c. Use the $>$, $<$, and $=$ symbols to compare two expressions involving addition and subtraction (e.g., $4 + 6 \square 3 + 2$; $3 + 5 \square 16 - 9$).
- d. Recognize and use the commutative, associative, distributive, and identity properties of addition and multiplication, and the zero property of multiplication.

Mathematical Language and Symbols Students Should Use

growing patterns, expressions, equations, $<$, $>$, $=$

Exploratory Concepts and Skills

- ✓ Use concrete materials to build an understanding of equality and inequality.
- ✓ Explore properties of equality in number sentences (e.g., when equals are added to equals, then the sums are equal; when equals are multiplied by equals, then the products are equal).

Standard 3: Students will describe and analyze attributes of two-dimensional shapes.

Objective 1: Describe and compare attributes of two-dimensional shapes.

- a. Identify, describe, and classify polygons (e.g., pentagons, hexagons, octagons).
- b. Identify attributes for classifying triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).
- c. Identify attributes for classifying quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).
- d. Identify right angles in geometric figures, or in appropriate objects, and determine whether other angles are greater or less than a right angle.

Objective 2: Demonstrate the meaning of congruence through applying transformations.

- a. Demonstrate the effect of reflection, translation, or rotation using objects.
- b. Determine whether two polygons are congruent by reflecting, translating, or rotating one polygon to physically fit on top of the other.

Mathematical Language and Symbols Students Should Use

polygon, attribute, quadrilateral, equilateral triangle, isosceles triangle, right triangle, pentagon, hexagon, octagon, parallel, right angle, reflect, translate, rotate, slide, flip, turn, congruent

Exploratory Concepts and Skills

- ✓ Explore line symmetry and rotational symmetry.
- ✓ Investigate two-dimensional representations of three-dimensional objects.

Standard 4: Students will select and use appropriate units and measurement tools to solve problems.

Objective 1: Select and use appropriate tools and units to estimate and measure length, weight, capacity, time, and perimeter of two-dimensional figures.

- a. Describe the part-whole relationships (e.g., 3 feet in a yard, a foot is $\frac{1}{3}$ of a yard) between metric units of length (i.e., centimeter, meter), and among customary units of length (i.e., inch, foot, yard), capacity (i.e., cup, quart), and weight (i.e., pound, ounce).
- b. Measure the length of objects to the nearest centimeter, meter, half- and quarter-inch, foot, and yard.
- c. Measure capacity using cups and quarts, and measure weight using pounds and ounces.
- d. Identify the number of minutes in an hour, the number of hours in a day, the number of days in a year, and the number of weeks in a year.
- e. Describe perimeter as a measurable attribute of two-dimensional figures, and estimate and measure perimeter with metric and customary units.

Objective 2: Solve problems involving measurements.

- a. Determine simple equivalences of measurements (e.g., 30 inches = 2 feet and 6 inches; 6 cups = $1\frac{1}{2}$ quarts; 90 min. = 1 hr. 30 min.).
- b. Compare given objects according to measurable attributes (i.e., length, weight, capacity).
- c. Solve problems involving perimeter.
- d. Determine elapsed time in hours (e.g., 7:00 a.m. to 2:00 p.m.).

Mathematical Language and Symbols Students Should Use

measure, unit, metric system, customary system, length, pound, ounce, centimeter, meter, inch, foot, yard, capacity, weight, perimeter

Exploratory Concepts and Skills

- ✓ Determine the value of a combination of coins and bills.
- ✓ Count back change from a single purchase.

Standard 5: Students will collect and organize data to make predictions and identify basic concepts of probability.

Objective 1: Collect, organize, and display data to make predictions.

- a. Collect, read, represent, and interpret data using tables, graphs, and charts, including keys (e.g., pictographs, bar graphs, frequency tables, line plots).
- b. Make predictions based on a data display.

Objective 2: Identify basic concepts of probability.

- a. Describe the results of events using the terms “certain,” “likely,” “unlikely,” and “impossible.”
- b. Conduct simple probability experiments, record possible outcomes systematically, and display results in an organized way (e.g., chart, graph).
- c. Use results of simple probability experiments to describe the likelihood of a specific outcome in the future.

Mathematical Language and Symbols Students Should Use

data, table, chart, graph, frequency table, line plot, pictograph, bar graph, likely, certain, outcome, impossible outcome

Exploratory Concepts and Skills

- ✓ Predict outcomes of simple experiments.

Fourth Grade Mathematics

By the end of grade four, students develop quick recall of the basic multiplication facts and related division facts. They develop fluency with efficient procedures for multiplying multidigit whole numbers, understand why the procedures work, and use them to solve problems. Students recognize decimal notation as an extension of the base-ten system. They relate their understanding of fractions to decimals. They generate equivalent fractions, simplify fractions, and identify equivalent fractions and decimals; compare and order whole numbers, simple fractions, and decimals to hundredths; and estimate decimal or fractional amounts in problem solving.

Students use transformations, including those that produce line and rotational symmetry. Students understand area as a measurable attribute of two-dimensional regions. They select appropriate units, strategies, and tools for solving problems that involve measuring area. They connect area measure to the area model for multiplication as a way to justify the formula for the area of a rectangle.

Standard 1: Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective 1: Demonstrate multiple ways to represent whole numbers and decimals, from hundredths to one million, and fractions.

- a. Read and write numbers in standard and expanded form.
- b. Demonstrate multiple ways to represent whole numbers and decimals by using models and symbolic representations (e.g., 36 is the same as the square of six, three dozen, or 9×4).
- c. Identify the place and the value of a given digit in a six-digit numeral, including decimals to hundredths, and round to the nearest tenth.
- d. Divide regions, lengths, and sets of objects into equal parts using a variety of models and illustrations.
- e. Name and write a fraction to represent a portion of a unit whole, length, or set for halves, thirds, fourths, fifths, sixths, eighths, and tenths.
- f. Identify and represent square numbers using models and symbols.

Objective 2: Analyze relationships among whole numbers, commonly used fractions, and decimals to hundredths.

- a. Compare the relative size of numbers (e.g., 475 is comparable to 500; 475 is small compared to 10,000 but large compared to 98).
- b. Order whole numbers up to six digits, simple fractions, and decimals using a variety of methods (e.g., number line, fraction pieces) and use the symbols $<$, $>$, and $=$ to record the relationships.
- c. Identify a number that is between two given numbers (e.g., 3.2 is between 3 and 4; find a number between 0.1 and 0.2).
- d. Identify equivalences between fractions and decimals by connecting models to symbols.
- e. Generate equivalent fractions and simplify fractions using models, pictures, and symbols.

Objective 3: Model and illustrate meanings of multiplication and division of whole numbers and the addition and subtraction of fractions.

- a. Model multiplication (e.g., equal-sized groups, rectangular arrays, area models, equal intervals on the number line), place value, and properties of operations to represent multiplication of a one- or two-digit factor by a two-digit factor and connect the representation to an algorithm.
- b. Use rectangular arrays to interpret factoring (e.g., find all rectangular arrays of 36 tiles and relate the dimensions of the arrays to factors of 36).
- c. Demonstrate the mathematical relationship between multiplication and division (e.g., $3 \times \square = 12$ is the same as $12 \div 3 = \square$ and $\square = 4$) and use that relationship to explain that division by zero is not possible.
- d. Represent division of a three-digit dividend by a one-digit divisor, including whole number remainders, using a variety of methods (e.g., rectangular arrays, manipulatives, pictures), and connect the representation to an algorithm.
- e. Use models to add and subtract simple fractions where one single-digit denominator is 1, 2, or 3 times the other (e.g., $2/4 + 1/4$; $3/4 - 1/8$).

Objective 4: Solve problems involving multiplication and division of whole numbers and addition and subtraction of simple fractions and decimals.

- a. Use estimation, mental math, paper and pencil, and calculators to perform mathematical calculations and identify when to use each one appropriately.
- b. Select appropriate methods to solve a single operation problem and estimate computational results or calculate them directly, depending on the context and numbers involved in a problem.
- c. Write a story problem that relates to a given multiplication or division equation, and select and write a number sentence to solve a problem related to the environment.
- d. Solve problems involving simple fractions and interpret the meaning of the solution (e.g., A pie has been divided into six pieces and one piece is already gone. How much of the whole pie is there when Mary comes in? If Mary takes two pieces, how much of the whole pie has she taken? How much of the pie is left?)

Objective 5: Compute problems involving multiplication and division of whole numbers and addition and subtraction of simple fractions and decimals.

- a. Demonstrate quick recall of basic multiplication and division facts.
- b. Multiply up to a three- digit factor by a two-digit factor with fluency, using efficient procedures.
- c. Divide up to a three-digit dividend by a one-digit divisor with fluency, using efficient procedures.
- d. Add and subtract decimals and simple fractions where one single-digit denominator is 1, 2, or 3 times the other (e.g., $2/4 + 1/4 = 3/4$; $1/3 - 1/6 = 1/6$).

Mathematical Language and Symbols Students Should Use

sum, difference, expanded form, standard form, square number, dividend, divisor, quotient, factor, product, array, multiple, numerator, denominator, sixths, eighths, tenths, equivalent, estimate, $<$, $>$, $=$, \neq

Exploratory Concepts and Skills

- ✓ Use concrete objects and visual models to add and subtract common decimals.
- ✓ Explore numbers less than zero by extending the number line and by using familiar applications such as temperature.
- ✓ Investigate the concept of ratio (e.g., the number of students to the number of teachers).

Standard 2: Students will use patterns and relations to represent mathematical problems and number relationships.

Objective 1: Identify, analyze, and determine rules for describing numerical patterns involving operations and nonnumerical growing patterns.

- a. Analyze growing patterns using objects, pictures, numbers, and tables to determine a rule for the pattern.
- b. Recognize, represent, and extend simple patterns involving multiples and other number patterns (e.g., square numbers) using objects, pictures, numbers, and tables.
- c. Identify simple relationships in real-life contexts and use mathematical operations to describe the pattern (e.g., the number of legs on a given number of chairs may be determined by counting by fours or by multiplying the number of chairs by 4).

Objective 2: Use algebraic expressions, symbols, and properties of the operations to represent, simplify, and solve mathematical equations and inequalities.

- a. Use the order of operations to evaluate, simplify, and compare mathematical expressions involving the four operations, parentheses, and the symbols $<$, $>$, and $=$ (e.g., $2 \times (4 - 1) + 3$; of the two quantities $7 - (3 - 2)$ or $(7 - 3) - 2$, which is greater?).
- b. Express single-operation problem situations as equations and solve the equation.
- c. Recognize that a symbol represents the same number throughout an equation or expression (e.g., $\Delta + \Delta = 8$; thus, $\Delta = 4$).
- d. Describe and use the commutative, associative, distributive, and identity properties of addition and multiplication, and the zero property of multiplication.

Mathematical Language and Symbols Students Should Use

growing pattern, order of operations, parentheses, inequality, expression, equation, associative property, commutative property, distributive property, zero property of multiplication, $>$, $<$, $=$

Exploratory Concepts and Skills

- ✓ Use concrete materials to build an understanding of equality and inequality.
- ✓ Explore properties of equality in number sentences (e.g., when equals are added to equals, then the sums are equal; when equals are multiplied by equals, then the products are equal).

Standard 3: Students will understand attributes and properties of plane geometric objects and spatial relationships.

Objective 1: Identify and describe attributes of two-dimensional geometric shapes.

- a. Name and describe lines that are parallel, perpendicular, and intersecting.
- b. Identify and describe right, acute, obtuse, and straight angles.
- c. Identify and describe the radius and diameter of a circle.
- d. Identify and describe figures that have line symmetry and rotational symmetry.

Objective 2: Specify locations using grids and maps.

- a. Locate coordinates in the first quadrant of a coordinate grid.
- b. Give the coordinates in the first quadrant of a coordinate grid.
- c. Locate regions on a map of Utah.
- d. Give the regions of a position on a map of Utah.

Objective 3: Visualize and identify geometric shapes after applying transformations.

- a. Identify a translation, rotation, or a reflection of a geometric shape.
- b. Recognize that 90° , 180° , 270° , and 360° are associated, respectively, with $1/4$, $1/2$, $3/4$, and full turns.

Mathematical Language and Symbols Students Should Use

parallel, perpendicular, intersecting lines, right angle, acute angle, obtuse angle, straight angle, circle, radius, diameter, line symmetry, rotational symmetry, coordinate, first quadrant, degree, translate, rotate, reflect, transformation

Exploratory Concepts and Skills

- ✓ Analyze results of transformations (e.g., translations, rotations, reflections) on two-dimensional shapes.
- ✓ Investigate two-dimensional representations of three-dimensional objects.

Standard 4: Students will describe relationships among units of measure, use appropriate measurement tools, and use formulas to find area measurements.

Objective 1: Describe relationships among units of measure for length, capacity, and weight, and determine measurements of angles using appropriate tools.

- a. Describe the relative size among metric units of length (i.e., millimeter, centimeter, meter), between metric units of capacity (i.e., milliliter, liter), and between metric units of weight (i.e., gram, kilogram).
- b. Describe the relative size among customary units of capacity (i.e., cup, pint, quart, gallon).
- c. Estimate and measure capacity using milliliters, liters, cups, pints, quarts, and gallons, and measure weight using grams and kilograms.
- d. Recognize that angles are measured in degrees and develop benchmark angles (e.g., 45° , 60° , 120°) using 90° angles to estimate angle measurement.
- e. Measure angles using a protractor or angle ruler.

Objective 2: Recognize and describe area as a measurable attribute of two-dimensional shapes and calculate area measurements.

- a. Quantify area by finding the total number of same-sized units of area needed to fill the region without gaps or overlaps.
- b. Recognize that a square that is 1 unit on a side is the standard unit for measuring area.
- c. Develop the area formula for a rectangle and connect it with the area model for multiplication.
- d. Develop and use the area formula for a right triangle by comparing with the formula for a rectangle (e.g., two of the same right triangles makes a rectangle).
- e. Develop, use, and justify the relationships among area formulas of triangles and parallelograms by decomposing and comparing with areas of right triangles and rectangles.
- f. Determine possible perimeters, in whole units, for a rectangle with a fixed area, and determine possible areas when given a rectangle with a fixed perimeter.

Mathematical Language and Symbols Students Should Use

millimeter, centimeter, meter, milliliter, liter, gram, kilogram, cup, pint, quart, gallon, area, perimeter

Exploratory Concepts and Skills

- ✓ Investigate perimeter of rectangles and squares.
- ✓ Investigate area of trapezoids.

Standard 5: Students will interpret and organize collected data to make predictions, answer questions, and describe basic concepts of probability.

Objective 1: Collect, organize, and display data to answer questions.

- a. Identify a question that can be answered by collecting data.
- b. Collect, read, and interpret data from tables, graphs, charts, surveys, and observations.
- c. Represent data using frequency tables, bar graphs, line plots, and stem and leaf plots.
- d. Identify and distinguish between clusters and outliers of a data set.

Objective 2: Describe and predict simple random outcomes.

- a. Describe the results of experiments involving random outcomes as simple ratios (e.g., 4 out of 9, $\frac{4}{9}$).
- b. Conduct simple probability experiments, with and without replacement, record possible outcomes systematically, and display results in an organized way.
- c. Use the results of simple probability experiments, with and without replacement, to describe the likelihood of a specific outcome in the future.

Mathematical Language and Symbols Students Should Use

data, line plot, line graph, bar graph, stem and leaf plot, cluster, outlier, frequency table, probability

Exploratory Concepts and Skills

- ✓ Explore minimum and maximum values for a set of data.
- ✓ Explore mean, median, mode, and range.

Fifth Grade Mathematics

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to whole numbers, fractions, and decimals. They locate integers on a number line and ordered pairs of integers on the coordinate plane. They determine rules for numerical patterns, work with expressions including order of operations, and solve single-operation equations involving a single variable. They classify angles, triangles, and quadrilaterals, and analyze relationships among lines, triangles and quadrilaterals. They recognize and determine surface area and volume of three-dimensional shapes, including right prisms. Students understand the concepts of mean, median, mode, and range of data sets and can calculate them. They use line plots, bar graphs, and line graphs to record and analyze data.

Standard 1: Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 1: Represent whole numbers and decimals from thousandths to one billion, fractions, percents, and integers.

- a. Read and write numbers in standard and expanded form.
- b. Demonstrate multiple ways to represent whole numbers, decimals, fractions, percents, and integers using models and symbolic representations (e.g., $108 = 2 \times 50 + 8$; $108 = 10^2 + 8$; $90\% = 90$ out of 100 squares on a hundred chart).
- c. Identify, read, and locate fractions, mixed numbers, decimals, and integers on the number line.
- d. Represent repeated factors using exponents.
- e. Describe situations where integers could be used in the students' environment.

Objective 2: Explain relationships and equivalencies among integers, fractions, decimals, and percents.

- a. Compare fractions by finding a common denominator.
- b. Order integers, fractions (including mixed numbers), and decimals using a variety of methods, including the number line.
- c. Rewrite mixed numbers and improper fractions from one form to the other and represent each using regions, sets of objects, or line segments.
- d. Represent commonly used fractions as decimals and percents in a variety of ways (e.g., models, fraction strips, pictures, calculators, algorithms).
- e. Model and calculate equivalent forms of a fraction (including simplest form).
- f. Rename whole numbers as fractions with different denominators (e.g., $5 = 5/1$, $3 = 6/2$, $1 = 7/7$).

Objective 3: Use number theory concepts to develop and use divisibility tests; classify whole numbers to 50 as prime, composite, or neither; and find common multiples and factors.

- a. Identify patterns with skip counting and multiples to develop and use divisibility tests for determining whether a whole number is divisible by 2, 3, 5, 6, 9, and 10.
- b. Use strategies for classifying whole numbers to 50 as prime, composite, or neither.
- c. Rewrite a composite number between 2 and 50 as a product of only prime numbers.
- d. Find common multiples and factors and apply to adding and subtracting fractions.

Objective 4: Model and illustrate meanings of multiplication and division.

- a. Represent division-with-remainder using whole numbers, decimals, or fractions.
- b. Describe the effect of place value when multiplying and dividing whole numbers and decimals by 10, 100, and 1,000.
- c. Model multiplication of fractions and decimals (e.g., tenths multiplied by tenths, a whole number multiplied by tenths, or a whole number with tenths multiplied by tenths) in a variety of ways (e.g., manipulatives, number line and area models, patterns).

Objective 5: Solve problems involving one or two operations.

- a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, and algorithms.
- b. Make reasonable estimations of fraction and decimal sums, differences, and products, including knowing whether results obtained using a calculator are reasonable.
- c. Write number sentences that can be used to solve a two-step problem.
- d. Interpret division-with-remainder problems as they apply to the environment (e.g., If there are 53 people, how many vans are needed if each van holds 8 people?).

Objective 6: Demonstrate proficiency with multiplication and division of whole numbers and compute problems involving addition, subtraction, and multiplication of decimals and fractions.

- a. Multiply multi-digit whole numbers by a two-digit whole number with fluency, using efficient procedures.
- b. Divide multi-digit dividends by a one-digit divisor with fluency, using efficient procedures.
- c. Add and subtract decimals with fluency, using efficient procedures.
- d. Add and subtract fractions with fluency.
- e. Multiply fractions.

Mathematical Language and Symbols Students Should Use

prime, composite, exponent, fractions, numerator, denominator, common denominator, common factor, common multiple, decimals, percents, divisible, divisibility, equivalent fractions, integer, dividend, quotient, divisor, factor, order of operations, simplest terms, various symbols for multiplication and division, mixed numeral, improper fraction

Exploratory Concepts and Skills

- ✓ Extend classification of whole numbers from 0-100 as prime, composite, or neither.
- ✓ Apply rules of divisibility.
- ✓ Explore adding and subtracting integers.
- ✓ Divide multi-digit dividends by a two-digit divisor.

Standard 2: Students will use patterns and relations to represent and analyze mathematical problems and number relationships using algebraic symbols.

Objective 1: Identify, analyze and determine a rule for predicting and extending numerical patterns involving operations whole numbers, decimals, and fractions.

- a. Analyze and make predictions about numeric patterns, including decimals and fractions.
- b. Determine a rule for the pattern using organized lists, tables, objects, and variables.

Objective 2: Use algebraic expressions, inequalities, or equations to represent and solve simple real-world problems.

- a. Use properties and the order of operations involving addition, subtraction, multiplication, division, and the use of parentheses to compute with whole numbers, decimals, and fractions.
- b. Use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities with whole number solutions (e.g., $6x = 54$; $x + 3 = 7$).

Mathematical Language and Symbols Students Should Use

variety of symbols for multiplication and division such as \times , \bullet , and $*$ as symbols for multiplication and \div , $\overline{)$, and a fraction bar ($/$ or $—$) as division symbols; variable, order of operations, parentheses, inequality, expression, equation, associative property, commutative property, distributive property

Exploratory Concepts and Skills

- ✓ Solve multi-step equations.
- ✓ Construct and analyze tables involving equivalent ratios.

Standard 3: Students will use spatial reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 1: Describe relationships between two- and three-dimensional shapes and analyze attributes and properties of geometric shapes.

- a. Draw, label, and describe line segments, rays, lines, parallel lines, and perpendicular lines.
- b. Draw, label, and define an angle as two rays sharing a common endpoint (vertex).
- c. Classify triangles and quadrilaterals and analyze the relationships among the shapes in each classification (e.g., a square is a rectangle).
- d. Relate pyramids and right prisms to the two-dimensional shapes (nets) from which they were created.
- e. Identify properties and attributes of solids (i.e., right prisms, pyramids, cylinders, cones) and describe them by the number of edges, faces, and vertices as well as the types of faces.

Objective 2: Specify locations in a coordinate plane.

- a. Locate points defined by ordered pairs of integers.
- b. Write an ordered pair for a point in a coordinate plane with integer coordinates.
- c. Specify possible paths between locations on a coordinate plane and compare distances of the various paths.

Mathematical Language and Symbols Students Should Use

perpendicular and parallel lines, rays, angles (acute, obtuse, right, straight), triangles (equilateral, isosceles, scalene, right, acute, obtuse), vertex, vertices, edge, face, corresponding angles, similar, polygon, pyramid, right prism

Exploratory Concepts and Skills

- ✓ Compare corresponding angles of two triangles and determine whether the triangles are similar.
- ✓ Rotate a shape around a fixed point and identify the location of the new vertices.
- ✓ Translate a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.
- ✓ Reflect a shape across either the x- or y-axis and identify the location of the new vertices.

Standard 4: Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Objective 1: Determine the area of polygons and apply to real-world problems.

- a. Determine the area of a trapezoid by the composition and decomposition of rectangles, triangles, and parallelograms.
- b. Determine the area of irregular and regular polygons by the composition and decomposition of rectangles, triangles, and parallelograms.
- c. Compare areas of polygons using different units of measure within the same measurement system (e.g., square feet, square yards).

Objective 2: Recognize, describe, and determine surface area and volume of three-dimensional shapes.

- a. Quantify volume by finding the total number of same-sized units of volume needed to fill the space without gaps or overlaps.
- b. Recognize that a cube having a 1 unit edge is the standard unit for measuring volume expressed as a cubic unit.
- c. Derive and use the formula to determine the volume of a right prism with a triangular or rectangular base.
- d. Relate the formulas for the areas of triangles, rectangles, or parallelograms to the surface area of a right prism.
- e. Derive and use the formula to determine the surface area of a right prism and express surface area in square units.

Mathematical Language and Symbols Students Should Use

area, volume, surface area, volume, right prism

Exploratory Concepts and Skills

- ✓ Investigate π as the ratio of the circumference to the diameter of a circle.
- ✓ Determine the volume of a right prism with various bases.

Standard 5: Students will construct, analyze, and construct reasonable conclusions from data and apply basic concepts of probability.

Objective 1: Formulate and answer questions using statistical methods to compare data, and propose and justify inferences based on data.

- a. Construct, analyze, and display data using an appropriate format (e.g., line plots, bar graphs, line graphs).
- b. Recognize the differences in representing categorical and numerical data.
- c. Identify minimum and maximum values for a set of data.
- d. Identify and calculate the mean, median, mode, and range.

Objective 2: Apply basic concepts of probability.

- a. Describe the results of experiments involving random outcomes using a variety of notations (e.g., 4 out of 9, $\frac{4}{9}$).
- b. Recognize that probability is always a value between 0 and 1 (inclusively).
- c. Express the likelihood of an outcome in a simple experiment as a value between 0 and 1 (inclusively).

Mathematical Language and Symbols Students Should Use

data, minimum values, maximum values, mean, median, mode, average, range

Exploratory Concepts and Skills

- ✓ Explore the differences in representing categorical and numerical data.

Sixth Grade Mathematics

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive rational numbers, positive decimals, and positive and negative integers; they accurately compute and solve problems. They find prime factorizations, least common multiples, and greatest common factors. They create, evaluate, and simplify expressions, and solve equations involving two operations and a single variable. They solve problems involving an unknown angle in a triangle or quadrilateral, and use properties of complementary and supplementary angles. Students know about π as the ratio between the circumference and the diameter of a circle and solve problems using the formulas for the circumference and area of a circle. Students analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Standard 1: Students will expand number sense to include operations with rational numbers.

Objective 1: Represent rational numbers in a variety of ways.

- a. Recognize a rational number as a ratio of two integers, a to b , where b is not equal to zero.
- b. Change whole numbers with exponents to standard form (e.g., $2^4 = 16$) and recognize that any non-zero whole number to the zero power equals 1 (e.g., $9^0 = 1$).
- c. Write a whole number in expanded form using exponents (e.g., $876,539 = 8 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$).
- d. Express numbers in scientific notation using positive powers of ten.

Objective 2: Explain relationships and equivalencies among rational numbers.

- a. Place rational numbers on the number line.
- b. Compare and order rational numbers, including positive and negative mixed fractions and decimals, using a variety of methods and symbols, including the number line and finding common denominators.
- c. Find equivalent forms for common fractions, decimals, percents, and ratios, including repeating or terminating decimals.
- d. Relate percents less than 1% or greater than 100% to equivalent fractions, decimals, whole numbers, and mixed numbers.
- e. Recognize that the sum of an integer and its additive inverse is zero.

Objective 3: Use number theory concepts to find prime factorizations, least common multiples, and greatest common factors.

- a. Determine whether whole numbers to 100 are prime, composite, or neither.
- b. Find the prime factorization of composite numbers to 100.
- c. Find the greatest common factor and least common multiple for two numbers using a variety of methods (e.g., list of multiples, prime factorization).

Objective 4: Model and illustrate meanings of operations and describe how they relate.

- a. Relate fractions to multiplication and division and use this relationship to explain procedures for multiplying and dividing fractions.
- b. Recognize that ratios derive from pairs of rows in the multiplication table and connect with equivalent fractions.
- c. Give mixed number and decimal solutions to division problems with whole numbers.

Objective 5: Solve problems involving multiple steps.

- a. Select appropriate methods to solve a multi-step problem involving multiplication and division of fractions and decimals.
- b. Use estimation to determine whether results obtained using a calculator are reasonable.
- c. Use estimation or calculation to compute results, depending on the context and numbers involved in the problem.
- d. Solve problems involving ratios and proportions.

Objective 6: Demonstrate proficiency with the four operations, with positive rational numbers, and with addition and subtraction of integers.

- a. Multiply and divide a multi-digit number by a two-digit number, including decimals.
- b. Add, subtract, multiply, and divide fractions and mixed numbers.
- c. Add and subtract integers.

Mathematical Language and Symbols Students Should Use

prime, composite, exponent, least common multiple, least common denominator, greatest common factor, decimals, percents, divisible, divisibility, equivalent fractions, integer, dividend, quotient, divisor, factor, simplest terms, mixed numeral, improper fraction

Exploratory Concepts and Skills

- ✓ Explore the addition and subtraction of positive and negative fractions.
- ✓ Investigate the concepts of ratio and proportion.
- ✓ Investigate the distributive property of multiplication over addition of double-digit multipliers.

Standard 2: Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 1: Analyze algebraic expressions, tables, and graphs to determine patterns, relations, and rules.

- a. Describe simple relationships by creating and analyzing tables, equations, and expressions.
- b. Draw a graph and write an equation from a table of values.
- c. Draw a graph and create a table of values from an equation.

Objective 2: Write, interpret, and use mathematical expressions, equations, and formulas to represent and solve problems that correspond to given situations.

- a. Solve single variable linear equations using a variety of strategies.
- b. Recognize that expressions in different forms can be equivalent and rewrite an expression to represent a quantity in a different way.
- c. Evaluate and simplify expressions and formulas, substituting given values for the variables (e.g., $2x + 4$; $x = 2$; therefore, $2(2) + 4 = 8$).

Mathematical Language and Symbols Students Should Use

order of operations, sequence, function, pattern, algebraic expression, approximately equal, \approx , notation for exponents: 4^3 or 4^3 , a number in front of a variable indicates multiplication (e.g., $3y$ means 3 times the quantity y), formula, generalization

Exploratory Concepts and Skills

- ✓ Use physical models to investigate and describe how a change in one variable affects a second variable.
- ✓ Use models to develop understanding of slope as constant rate of change.
- ✓ Model situations with proportional relationships and solve problems.

Standard 3: Students will use spatial and logical reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 1: Identify and analyze attributes and properties of geometric shapes to solve problems.

- a. Identify the midpoint of a line segment and the center and circumference of a circle.
- b. Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.
- c. Develop and use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle in a triangle or quadrilateral.

Objective 2: Visualize and identify geometric shapes after applying transformations on a coordinate plane.

- a. Rotate a polygon about the origin by a multiple of 90° and identify the location of the new vertices.
- b. Translate a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.
- c. Reflect a polygon across either the x- or y-axis and identify the location of the new vertices.

Mathematical Language and Symbols Students Should Use

midpoint, circumference, complementary and supplementary angles, rotate, translate, reflect, transformation

Exploratory Concepts and Skills

- ✓ Use manipulatives and technology to model geometric shapes.
- ✓ Investigate tessellations.
- ✓ Explore the angles formed by intersecting lines.
- ✓ Identify and draw shapes and figures from different views/perspectives.

Standard 4: Students will understand and apply measurement tools and techniques and find the circumference and area of a circle.

Objective 1: Describe and find the circumference and area of a circle.

- a. Explore the relationship between the radius and diameter of a circle to the circle's circumference to develop the formula for circumference.
- b. Find the circumference of a circle using a formula.
- c. Describe pi as the ratio of the circumference to the diameter of a circle.
- d. Decompose a circle into a number of wedges and rearrange the wedges into a shape that approximates a parallelogram to develop the formula for the area of a circle.
- e. Find the area of a circle using a formula.

Objective 2: Identify and describe measurable attributes of objects and units of measurement, and solve problems involving measurement.

- a. Recognize that measurements are approximations and describe how the size of the unit used in measuring affects the precision.
- b. Convert units of measurement within the metric system and convert units of measurement within the customary system.
- c. Compare a meter to a yard, a liter to a quart, and a kilometer to a mile.
- d. Determine when it is appropriate to estimate or use precise measurement when solving problems.
- e. Derive and use the formula to determine the surface area and volume of a cylinder.

Mathematical Language and Symbols Students Should Use

cylinder, radius, diameter, circumference, area, surface area, volume, π

Exploratory Concepts and Skills

- ✓ Investigate volumes and surface areas of a variety of three-dimensional objects.

Standard 5: Students will analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Objective 1: Design investigations to reach conclusions using statistical methods to make inferences based on data.

- a. Design investigations to answer questions.
- b. Extend data display and comparisons to include scatter plots and circle graphs.
- c. Compare two similar sets of data on the same graph and compare two graphs representing the same set of data.
- d. Recognize that changing the scale influences the appearance of a display of data.
- e. Propose and justify inferences and predictions based on data.

Objective 2: Apply basic concepts of probability and justify outcomes.

- a. Write the results of a probability experiment as a fraction between zero and one, or an equivalent percent.
- b. Compare experimental results with theoretical results (e.g., experimental: 7 out of 10 tails; whereas, theoretical 5 out of 10 tails).
- c. Compare individual, small group, and large group results of a probability experiment in order to more accurately estimate the actual probabilities.

Mathematical Language and Symbols Students Should Use

data display, scatter plot, circle graph, scale, predict, justify, probability, experimental results, theoretical results

Exploratory Concepts and Skills

- ✓ Investigate the notion of fairness in games.